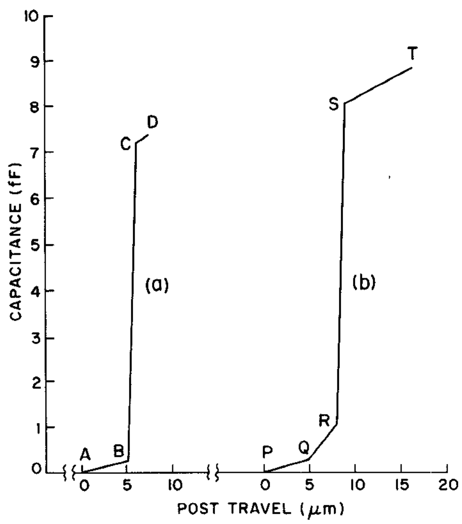

Fig. 3. Capacitance measured at the IF connector during contacting. In curve (a) the whisker approaches the diode (AB), contacts its anode (BC), and deforms slightly (CD) as the post is advanced 1–2 μm further to ensure a stable contact. In curve (b) the whisker lands on the SiO_2 layer between diodes (Q), is forced to skid along the SiO_2 with considerable deformation (QR) until it hits an anode (RS), and is then further deformed (ST) by excessive additional advancement of the post.

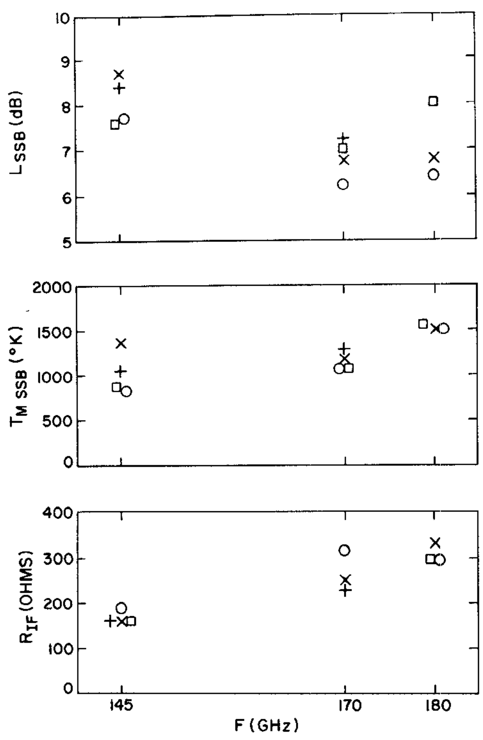

Fig. 4. Measured SSB conversion loss L, mixer noise temperature T_M, and IF impedance R_{IF} for several mixers. L and T_M are referred to the mixer input flange, and have been corrected for IF mismatch. T_M does not include IF amplifier noise. R_{IF} is deduced from the measured IF-port VSWR. Optimum bias was ~0.6 V with $I_{dc} \approx 1.0$ mA.

against hot and cold loads, enables the conversion loss and mixer noise temperature to be determined with respect to the waveguide input flange of the mixer. An IF reflectometer gives the VSWR at the IF port. The LO was injected through a tunable resonant-ring filter having a LO noise rejection of typically 14 dB at the signal and image frequencies (1.4 GHz either side of the LO). For some klystrons the (AM) sideband noise can be many times greater than the input noise temperature of the mixer, and even with 14 dB of sideband rejection the klystron noise may severely degrade the system noise temperature.[1] Our results have been corrected for klystron noise and IF mismatch, and give the performance which would be obtained using a quiet (or well filtered) LO, and an appropriate IF transformer.

In Fig. 4 the measured performance of four mixers is shown. L_{SSB} is the single-sideband conversion loss, and T_M is the effective SSB input noise temperature of the mixer excluding noise from the IF amplifier. The differences between the results for these mixers are caused by variations of diode parameters and mount dimensions. The optimum bias was found to be ~0.6 V with ~1.0-mA dc current. The LO power requirement is estimated to be 1–3 mW. (No power meter was available, and this estimate is based on the klystron manufacturer's figures for our tubes.)

V. Conclusions

A new Schottky-diode mixer has been demonstrated which has low conversion loss and noise in the 140–220-GHz waveguide band. The mixer has an asymmetrical split-block configuration which should be usable up to 300 GHz with appropriate scaling. The diode chip is recessed into the waveguide wall and forms part of the RF choke structure. An improved diode contacting procedure is used.

Acknowledgment

The authors wish to thank I. Silverberg and H. Miller who fabricated the mixer blocks, and G. Green who made the diodes.

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The Measurement of the Equivalent Admittance of 3-Port Circulators Via an Automated Measurement System

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***Abstract*—A derivation of the equivalent admittance of symmetrical 3-port circulators is given which is based on the requirement that the S-matrix eigenvalues be separated by 120° on the unit circle for perfect circulation. This quantity has the property that if a 2-port matching network is found which matches into this admittance, then the same matching network connected in each circulator arm will match the circulator. It was used in conjunction with a computerized measurement system to determine the optimal single-step transformer matching of a stripline 3.7–4.2-GHz circulator and resulted in a device with performance better than 30 dB over this band.**

Manuscript received March 11, 1976; revised August 23, 1976.
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[1] For example, our 180-GHz tube, when used as a LO with no sideband filtering, adds about 20 000 K to the system temperature, while our 170-GHz tube only adds 3 000 K. With an LO filter having a sideband rejection of 14 dB, the corresponding contributions to T_{sys} are 800 and 120 K.

I. Introduction

In the past various techniques have been proposed for determining the identical 2-port matching networks which must be connected in each arm of a symmetrical 3-port circulator in order to match the device. One of the most frequent techniques is to insert a double stub tuner in port 2 and tune it for minimum power out of port 3 [1]. The input admittance at port 1 is then the admittance into which the matching network must match. This sort of approach has been discussed in some detail by Bosma [2]. The disadvantage of such an approach is that it requires some sort of a mechanical tuner and hence cannot be directly employed in a computerized measurement system. Expressions for this input admittance, which is sometimes called the complex gyrator admittance, are known from the literature [3]–[5]. A different derivation will be given here which is based on the requirement that the S-matrix eigenvalues be separated by 120° on the unit circle for perfect circulation. The resulting quantity is an algebraic function of the 3 eigenadmittances Y_0, Y_{-1}, and Y_1, and can readily be determined using a computerized measurement system.

II. The Equivalent Admittance at a Reference Plane with $jY_0 = j\infty$

Suppose we consider a symmetrical nonreciprocal lossless 3-port network with eigenadmittances Y_0, Y_{-1}, and Y_1 and connect identical matching networks at each port. If Y_0', Y_{-1}', and Y_1' are the eigenadmittances of the symmetrical device which includes the matching networks, then

$$jY_i' = j\frac{C + DY_i}{A - BY_i}, \qquad i = 0, -1, 1 \tag{1}$$

where A, B, C, and D are the elements of the $ABCD$ matrix of the matching network as in Fig. 1. It will be recalled that the condition for perfect circulation is that the three phases of the eigenreflection coefficients after matching ψ_0', ψ_{-1}', and ψ_1' differ by 120°.

$$\therefore \psi_1' = \psi_0' + \frac{2\pi}{3} \quad \psi_{-1}' = \psi_0' - \frac{2\pi}{3}. \tag{2}$$

Since $jY_i' = j\tan(-\psi_i'/2)$, the required conditions on the eigenadmittances can be obtained by taking the tangent of both sides of (2). These conditions are

$$-Y_{+1}' = \frac{-Y_0' + \sqrt{3}}{1 + \sqrt{3}\,Y_0'} \quad -Y_{-1}' = \frac{-Y_0' - \sqrt{3}}{1 - \sqrt{3}\,Y_0'}. \tag{3}$$

Substituting (1) into (3), the conditions for a perfect match become

$$-\frac{C + DY_1^*}{A - BY_1^*} = \frac{D + \sqrt{3}\,B}{B - \sqrt{3}\,D}$$

$$-\frac{C + DY_{-1}^*}{A - BY_{-1}^*} = \frac{D - \sqrt{3}\,B}{B + \sqrt{3}\,D} \tag{4}$$

where we have specialized to reference planes with $jY_0 = j\infty$. Starred quantities will refer to such reference planes which can always be found. Multiplying out denominators, one finds that

$$AD + BC - \sqrt{3}\,(B^2 + D^2)Y_1^* = -\sqrt{3}\,(AB - CD)$$

$$AD + BC + \sqrt{3}\,(B^2 + D^2)Y_{-1}^* = \sqrt{3}\,(AB - CD). \tag{5}$$

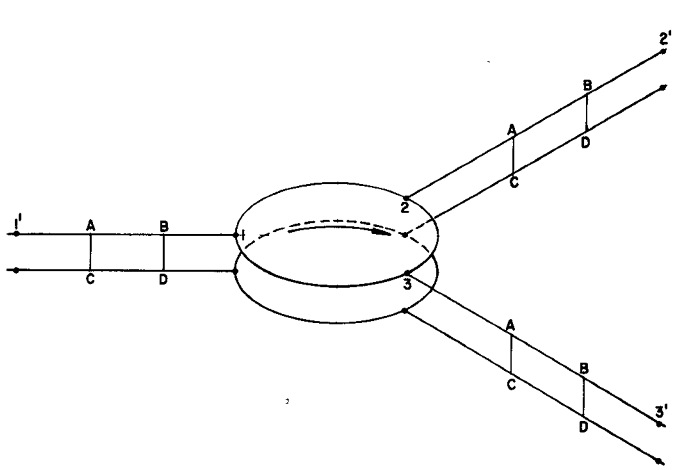

Fig. 1. A symmetrical nonreciprocal 3-port network with identical 2-port matching networks connected in each arm. Unprimed quantities refer to those of the basic junction while primed quantities refer to those obtained after matching.

Setting $AD + BC = 1$ for reciprocal matching networks, then adding and subtracting both sides of (5), the following two equations result:

$$(B^2 + D^2)\left\{\frac{Y_1^* + Y_{-1}^*}{2}\right\} = AB - CD$$

$$(B^2 + D^2)\left\{\frac{(Y_1^* - Y_{-1}^*)\sqrt{3}}{2}\right\} = 1. \tag{6}$$

These equations should look familiar. These circulator matching conditions are precisely the conditions for matching into the complex admittance $G^* + jY^*$ provided the following identifications are made:

$$G^* = \frac{Y_1^* - Y_{-1}^*}{2}\sqrt{3} \quad Y^* = \frac{Y_1^* + Y_{-1}^*}{2}. \tag{7}$$

The input admittance Y_{in} of the matching network terminated in the admittance $G^* + jY^*$ is

$$Y_{\text{in}} = \frac{jC + D(jY^* + G^*)}{A + jB(jY^* + G^*)}.$$

A perfect match implies that $Y_{\text{in}} = 1$ or that

$$(B^2 + D^2)G^* = (AD + BC) = 1$$

$$(B^2 + D^2)Y^* = AB - CD.$$

These equations are of the same form as that of (6) and are identical to it if the identifications (7) are made. The expressions for the real and imaginary parts of the equivalent admittance assume a particularly simple form at reference planes where $jY_0 = j\infty$. A similar calculation has been carried out by Helszajn, who assumes a circulator model with $\psi_0 = \pi$ [6].

III. The Equivalent Admittance at an Arbitrary Reference Plane

To obtain the admittance at an arbitrary reference plane where the symmetrical mode phase is ψ_0, one must transform the admittance $G^* + jY^*$ through a length of transmission line of electrical length $\theta = -(\psi_0 - \pi)/2$. Such a transformation takes the phase of the symmetrical eigenexcitation from π to ψ_0.

$$\therefore\ Y_{eq} = \frac{(G^* + jY^*)\cos\theta + j\sin\theta}{\cos\theta + j\sin\theta(G^* + jY^*)}. \tag{8}$$

Expressions for G^* and Y^* in terms of the eigenadmittances at an arbitrary reference plane can be found by using (7) and by finding expressions for $Y_1{}^*$ and $Y_{-1}{}^*$ in terms of these eigenadmittances. Now,

$$jY_1{}^* = j\tan\left(-\frac{\psi_1 - \psi_0 - \pi}{2}\right)$$

$$jY_{-1}{}^* = j\tan\left(-\frac{\psi_{-1} - \psi_0 - \pi}{2}\right). \tag{9}$$

The desired expressions can now be found if the tangent functions are expanded using the appropriate trigonometric identity and use is made of the relations $Y_i = j\tan(-\psi_i/2)$ and $i = 0, -1, 1$. The phase angle θ can be eliminated from (8) by dividing numerator and denominator by $\cos\theta$ and finding an expression for $\tan\theta$ in terms of Y_0 in a similar way. The algebraic manipulations are straightforward. The result for the real and imaginary parts is

$$G = \frac{(1 + Y_0{}^2)G^*}{G^{*2} + (Y^* + Y_0)^2}$$

$$Y = \frac{G^{*2}Y_0 - (1 - Y^*Y_0)(Y^* + Y_0)}{G^{*2} + (Y^* + Y_0)^2} \tag{10}$$

where

$$G^* = \frac{\sqrt{3}}{2}\left\{\frac{Y_1 + 1/Y_0}{1 - Y_1/Y_0} - \frac{Y_{-1} + 1/Y_0}{1 - Y_{-1}/Y_0}\right\}$$

$$Y^* = \frac{1}{2}\left\{\frac{Y_1 + 1/Y_0}{1 - Y_1/Y_0} + \frac{Y_{-1} + 1/Y_0}{1 - Y_{-1}/Y_0}\right\}. \tag{11}$$

It is well known that ψ_0, ψ_1, and ψ_{-1} and hence Y_0, Y_1, and Y_{-1} can be determined on certain computerized measurement systems [7]. Consequently, the circulator equivalent admittance can readily be evaluated on such systems.

The existence of expressions (10) and (11) allows certain well-known propositions about 3-port circulators to be derived. If the device is reciprocal so that $Y_1 = Y_{-1}$, then $G^* = 0$ from (11). Likewise $G = 0$ from (10) and it is impossible to match the device. This is the usual result. Otherwise G and Y will be some real numbers. Consequently, except at isolated frequencies where G may be 0 or ∞ or Y may be $\pm\infty$, any symmetrical nonreciprocal 3-port device can be matched to be a circulator. If G is negative, then port 2 will be the isolated port instead of port 3 as was implicitly assumed by the choice of the eigenvalue phase relations of (2). A positive value for G is obtained by interchanging the roles of ψ_1 and ψ_{-1} or equivalently port 2 with port 3.

IV. The Measurement of the Equivalent Admittance

The preceding admittance can readily be measured with the Hewlett Packard computerized measurement system. The technique which must be used has been suggested by Knerr [7]. Ports 1 and 2 are connected as if to measure the S-matrix parameters of a 2-port network, and port 3 is terminated in a load. Clearly, the parameters S_{11} and S_{12} can be found in this way. The third parameter S_{13} can also be found since $S_{13} = S_{21}$ by symmetry and S_{21} is one of the 2-port parameters. The eigenreflection coefficients S_0, S_{-1}, and S_1 and the eigenadmittances are readily found now using standard formulas. From these the equivalent admittance can be calculated.

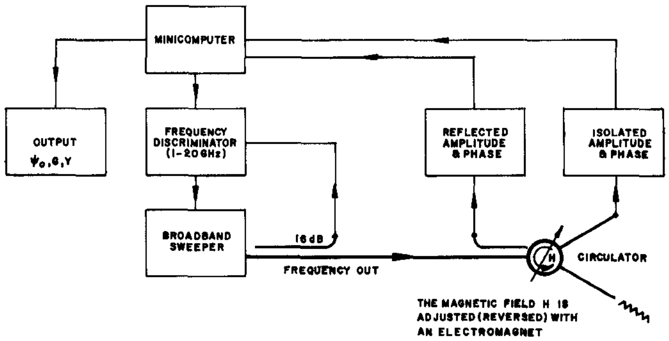

Fig. 2. A schematic diagram of the experimental arrangement used to measure the equivalent admittance of a coax circulator junction. The magnetic field on the circulator is varied (reversed) by varying (reversing) the coil current of an electromagnet.

Unfortunately, this measurement system was not available for our use. Instead a computerized system which we have been developing for making automated measurements on multiport waveguide devices was modified for the purpose. A schematic diagram of the experimental arrangement is given in Fig. 2. A solid-state signal source is controlled by a YIG tuned frequency discriminator.[1] This has proven to be a quite satisfactory method of frequency control to within 1 MHz. The reflected amplitude and phase at port 1 and transmitted amplitude and phase at port 2 are inputed to a minicomputer. Port 3 is terminated in a load. The magnetic field on the circulator is controlled with an electromagnet. To determine the parameter S_{13}, the current through the electromagnet is reversed and the new transmitted amplitude and phase are measured. With these parameters the minicomputer is able to calculate and print out the phase of the symmetrical mode ψ_0 and the real and imaginary parts of the equivalent admittance G and Y, respectively, as a function of frequency.

Measurements as a function of frequency were carried out for various magnetic field strengths on a basic stripline circulator junction. The two garnet disks were chosen to have a radius such that the first dielectric resonant frequency was centered in the 3.7–4.2-GHz band. Their diameter was slightly larger than that of the center conductor disk. The three air lines leading up to the disks had an impedance of 50 Ω. The reference plane was set at the edge of the garnet boundary.

In Table I the values of ψ_0 (the phase of the symmetrical mode) and the real and imaginary parts of the admittance G and Y are given as a function of frequency for various magnetic field levels. One sees that in the chosen band 3.5–4.3 GHz ψ_0 is, except for a slight slope versus frequency, nearly constant at 180° independent of the magnetic field level used. This agrees with certain circulator models which assume the symmetrical mode to be short circuited at the ferrite boundary [6]. The conductance is nearly independent of frequency depending essentially only on the applied magnetic field. The susceptance Y has a nearly linear slope versus frequency, the slope of which decreases as the coil current increases.

In Fig. 3 the conductance G is plotted versus the coil current of the electromagnet. The conductance increases linearly in current up to about 1 A, while above this current level the conductance starts to saturate to a value slightly larger than 3. At 2.5 A the conductance was 3.25. This is in spite of the fact that the magnetic field strength between the poles of the electromagnet was measured to continue to increase linearly with current. In Fig. 4 the susceptance slope parameter B determined from the susceptance data is plotted versus conductance. One has the result that

[1] Available from NYTEK Electronics, Palo Alto, CA.

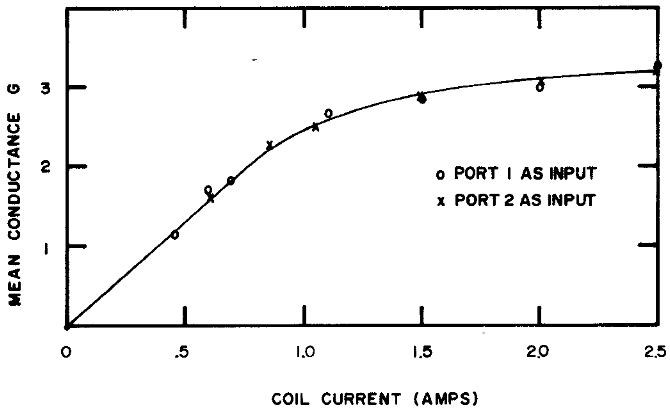

Fig. 3. A plot of the conductance G versus coil current. The conductance saturates for large currents and is 3.25 for $i = 2.5$ A.

TABLE I
PHASE OF THE SYMMETRICAL MODE ψ_0, CONDUCTANCE G, AND ADMITTANCE Y AS DETERMINED BY THE COMPUTERIZED MEASUREMENT SYSTEM FOR A STRIPLINE CIRCULATOR JUNCTION

	Coil current = o			Coil Current = 450 ma		
FREQ.(GHz)	ψ_0	G	Y	ψ_0	G	Y
3500	183.2	0	-1.387	182.5	1.354	-1.221
3600	180.9	0	-1.056	181.1	1.281	-.934
3700	179.1	0	-.734	180.0	1.211	-.606
3800	178.2	0	-.437	180.0	1.171	-.319
3900	177.9	.002	-.138	180.4	1.152	-.044
4000	177.0	-.003	.146	179.6	1.139	.206
4100	175.1	-.002	.413	176.7	1.126	.410
4200	174.7	.002	.691	173.8	1.096	.625
4300	175.7	.003	.975	172.2	1.056	.878

	Coil current = 700 ma			Coil current = 1050 ma		
3500	181.2	1.934	-1.035	183.4	2.441	-.695
3600	181.4	1.843	-.730	184.2	2.420	-.412
3700	182.4	1.805	-.432	184.5	2.413	-.159
3800	183.3	1.797	-.142	184.9	2.447	.091
3900	183.5	1.798	.115	184.8	2.445	.323
4000	182.4	1.795	.325	183.5	2.469	.463
4100	179.5	1.841	.483	182.4	2.499	.637
4200	176.3	1.856	.623	180.5	2.521	.759
4300	172.9	1.848	.764	178.0	2.540	.831

	Coil current = 1.5 amps			Coil current = 2 amps		
FREQ.(GHz)	ψ_0	G	Y	ψ_0	G	Y
3500	188.3	2.836	-.290	187.8	3.057	-.410
3600	188.9	2.805	-.020	188.1	3.037	-.156
3700	189.4	2.801	.240	188.5	2.989	.104
3800	189.9	2.808	.512	188.8	2.996	.350
3900	189.8	2.795	.724	188.7	2.982	.568
4000	188.9	2.801	.864	188.0	3.015	.730
4100	187.2	2.872	.994	186.5	3.053	.847
4200	186.1	2.944	1.144	184.8	3.098	.910
4300	185.3	2.916	1.267	182.3	3.148	.875

Note: Given for the frequency range 3.5–4.3 GHz for six different coil currents of the electromagnet.

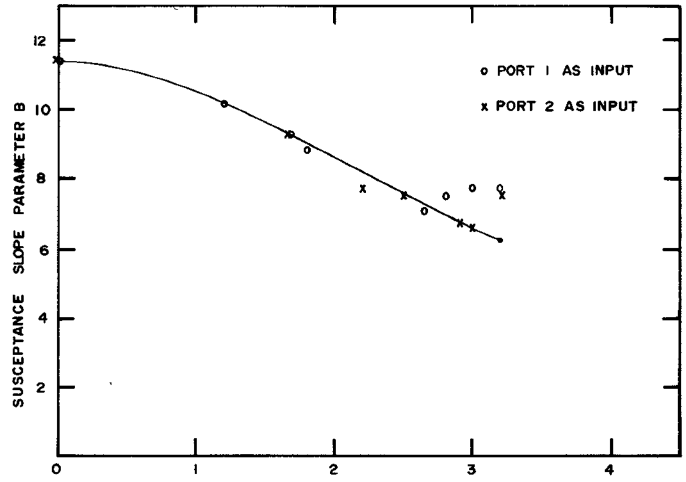

Fig. 4. A plot of the susceptance slope parameter B as determined from the data of Table I versus conductance G. B decreases as the conductance saturates.

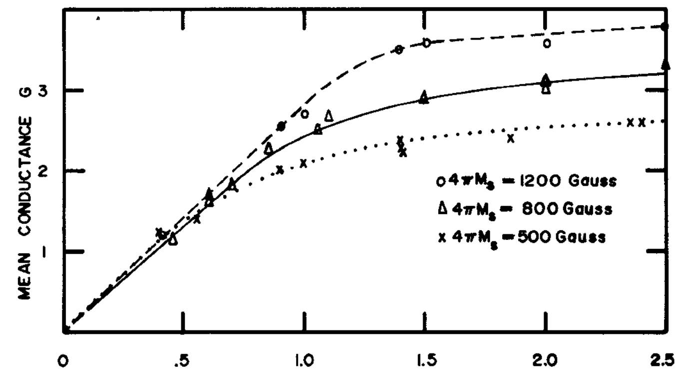

Fig. 5. A plot of conductance G versus coil current for garnet disks with magnetizations of 500, 800, and 1200 G, respectively. The maximum conductance $G_{\max}$ increases with magnetization.

B decreases as G increases. In summary, these experimental results confirm that the equivalent admittance is that of a shunt resonator with the important addition that the susceptance slope parameter B of the resonator is a function of the conductance G and decreases as the conductance approaches its saturation value.

In Fig. 5 a plot is given of the mean conductance versus coil current for garnet disks of the same size but different magnetizations of 500, 800, and 1200 G, respectively. The initial slope is the same but the maximum conductance $G_{\max}$ in the region of saturation increases with magnetization. Whereas $G_{\max}$ for the garnet disks with magnetization of 1200 was about 4, $G_{\max}$ for ferrite disks of magnetization of 1300 was about 3. This fact is presumably related to the lower dielectric constant of ferrites. It was also observed that the negative slope of the curve $B(G)$ increases significantly with increased magnetization in agreement with the observations of Simon [1].

The 1-port shunt resonator model has long been used to determine the optimum matching of 3-port circulators. Various authors have shown how single- or multiple-step quarter-wavelength transformers may be used to broad band a shunt resonator [8]–[10]. A similar procedure will be carried through here to determine the required single-step transformer to optimize the performance over the 3.7–4.2-GHz band. The conditions which must be satisfied for a perfect match are

$$\left\{\frac{1}{Y^2}\sin^2\frac{\pi}{2}(1+\Delta)+\cos^2\frac{\pi}{2}(1+\Delta)\right\}G=1$$

$$\left\{\frac{1}{Y^2}\sin^2\frac{\pi}{2}(1+\Delta)+\cos^2\frac{\pi}{2}(1+\Delta)\right\}B(G)\Delta$$
$$=\sin\frac{\pi}{2}(1+\Delta)\cos\frac{\pi}{2}(1+\Delta)\left\{\frac{1}{Y}-Y\right\} \quad (12)$$

where Y is the transformer admittance level, G is the equivalent conductance, $B(G)$ is the susceptance slope parameter, and $\Delta = (\omega - \omega_0)/\omega_0$ with ω_0 being the circulator center frequency and ω the frequency for which the device should be matched. These conditions are obtained from (6) and (7) if the *ABCD* matrix elements of a single-step transformer are substituted. Notice that these equations remain unchanged if Δ is replaced by $-\Delta$. This means that there will be two frequencies of perfect match symmetrically located about the center frequency. If we choose $\Delta = 0.0447$ so as to have optimum performance over the 3.7–4.2 band, then (12) constitutes two equations in two unknowns Y and G. The dependence of B upon G introduces a complication in their solution. In practice, they were solved by assuming values for B and then solving for Y and G until the value of B obtained from the curve of Fig. 4 for the calculated value of G agreed with assumed value. The result for $\Delta = 0.0447$ was $B = 6.3$, $Y = 1.805$, and $G = 3.217$. The theoretical VSWR is less than 1.01 over the band, which obviously will not be obtainable in practice because of irreproducible connector mismatches. Transformer dielectrics with a dielectric constant giving an admittance level close to 1.80 were now inserted in each arm of the circulator. The resulting device had a performance better than 30 dB over the 3.7–4.2 band.

V. Conclusions

The equivalent or complex gyrator admittance of a standard stripline circulator junction has been measured with a computerized measurement system as a function of frequency and magnetic field. The results confirm that this admittance is that of a shunt resonator close to the first dielectric resonant frequency of the garnet disks. Several interesting features did emerge. The conductance initially increases linearly with magnetic field but then begins to saturate approaching a saturation value of 3.25. The susceptance slope parameter B of the resonator is a function of the magnetic field and decreases in the region where the conductance is saturating. The data were used to determine the matching transformer required to build a device with better than 30 dB return loss over the 3.7–4.2-GHz band.

Acknowledgment

The author wishes to thank D. Hagerman and J. Madden for diligent efforts in getting the computerized measurement system in operation.

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Dimensions of Microstrip Coupled Lines and Interdigital Structures

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Abstract—**A method is presented for finding dimensions of coupled lines and interdigital structures on microstrip given the electrical properties. Both a graphical approach and computer approach using polynomial approximations are given. These results are within 1 percent of Bryant and Weiss' dimensions for coupled lines for most practical stripwidths and spacings. Experimental data for a 10-percent bandwidth microstrip interdigital filter are given.**

I. Introduction

The design of microwave filters and couplers on microstrip requires data on coupled lines and interdigital structures. The design of these structures has been done using tables or graphs generated from the work of Bryant and Weiss [1], Smith [2], or others. Until now there has been no method of designing microstrip interdigital structures. All of these methods start with dimensions and end with the electrical properties of the structure. This short paper gives a method of obtaining dimensions of the lines from the self and mutual capacitances. The method is presented in both a graphical form and polynomial approximations which can be programmed. Coupled lines with unequal linewidths and interdigital structures can be approximated.

The curves obtained here follow the same idea as Cristal's [5] method for coupled rods between ground planes. Cristal derived the curves by analyzing a periodic structure of equal rods and devised an approximation method to find interdigital structure dimensions with these curves. The same idea can be extended to microstrip with a few minor changes. It should be pointed out that this method can find stripwidths and spacings, but it does not handle the problem of finding the velocity of the waves on the lines. For N lines in an inhomogeneous structure such as microstrip, there are N possible normal modes with, in general, N different velocities. Interdigital and combline filters do not fit the normal modes, and it is a problem determining the proper length to make the lines. The proper resonator length of even the simpler coupled line filters is difficult to determine. The method was tried experimentally on a five-section interdigital filter and shows usable results even though the filter deviates from the theoretical response. Section II covers the derivation and use of the curves, Section III gives polynomial approximations of the curves and explains their use, and Section IV reports on an experimental filter.

Manuscript received October 13, 1975; revised August 23, 1976.
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